Modeling Aerodynamic Responses to Aircraft Maneuvers – A Numerical Validation

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The regime of validity of an aerodynamic mathematical model, applicable to describe the nonlinear aerodynamic reactions to a delta wing maneuvering at high angles of attack, is investigated. An unsteady vortex-lattice method is used to compute the unsteady flowfields, and thus to evaluate the aerodynamic data required by the model, in terms of specified characteristic motions. Time-histories of the aerodynamic responses to complex motions are generated by means of the model and the evaluated aerodynamic data and are compared with baseline aerodynamic responses obtained from direct vortex-lattice computations. The validity of the mathematical modeling approach for the maneuvering delta wing is demonstrated by agreement of the force and moment responses obtained from the two approaches.

Introduction

REQUIREMENTS imposed on modern aircraft and missiles for increased performance and maneuverability have resulted in extending their flight envelopes into the high-angle-of-attack regime. Aircraft maneuvering in this regime are subject to nonlinear, unsteady aerodynamic loads. The nonlinearities and unsteadiness are due mainly to the large regions of three-dimensional separated flow and concentrated vortex flows that occur at large angles of attack and, in the case of aircraft maneuvering at transonic speeds, to the presence and movement of shock waves. Accurate prediction of these nonlinear, unsteady airloads is of great importance in the analysis of a vehicle's flight motions and in the design of its flight control system. Prediction of the unsteady airloads is complicated by the fact that the instantaneous flowfield surrounding a maneuvering body, and thus the loading, is not determined solely by the instantaneous values of the motion variables, such as the angles of attack and sideslip and the control deflection angles. In general, the instantaneous state of the flowfield depends on the time-history of the motion, that is, on all the states taken by the flowfield during the maneuver prior to the instant in question.

Today, in light of the remarkable advances in computer technology and numerical algorithms for the computation of fluid flowfields, we can envision utilizing the computer to overcome the difficulty of accounting for time-history effects in determining the aerodynamic reactions (cf. Ref. 1 for a recent review of computational capabilities). For example, if codes and closure models adequate to solve the time-dependent Reynolds-averaged form of the Navier-Stokes equations governing the unsteady separated flowfield surrounding an aircraft are assumed available, a straightforward approach to accounting for time-history effects would be to solve the flowfield equations simultaneously with the dynamic equations governing the vehicle's motion. Such a coupled approach is shown schematically in Fig. 1a. Results of these coupled computations would be complete time histories of the aerodynamic response and of the vehicle motion. It is noteworthy that computations involving coupled equations have recently been carried out for several unsteady two-dimensional inviscid flows[2-4] and for at least two cases involving unsteady two-dimensional viscous flows by means of the time-dependent Reynolds-averaged form of the Navier-Stokes equations.[5,6] However, lack of computational resources has, to date, precluded undertaking the analogous three-dimensional viscous computations.

Although coupling of the flowfield equations and the aircraft's inertial equations of motion is, in principle, an exact way of accounting for time-history effects in predicting the aerodynamic response to arbitrary maneuvers, it inevitably will be a very costly approach. This will be particularly true for manuevers at high incidence, where the airloads depend nonlinearly on the motion variables. Under such conditions the aircraft can experience widely varying motion histories, even if they are started from closely spaced initial conditions. Thus, to evaluate an aircraft's performance envelope completely, a large number of computational cases, each involving the coupled equations, would be required to cover all possible sets of initial conditions. Since the motion and the aerodynamic responses are inextricably linked in the coupled approach, the flowfields must be recomputed for each change in initial conditions.

An alternative approach, one that eliminates the necessity of recomputing the flowfields, relies on mathematical modeling to describe the steady and unsteady aerodynamic terms in the aircraft's equations of motion. The modeling approach is shown schematically in Fig. 1b. In formulating a model, one attempts to specify a *form* for the aerodynamic response that remains the same in the determination of the aerodynamic response to all motions of interest. Ideally, with a mathematical model, an evaluation of the aerodynamic terms specified by the model would be required only once, whereupon they could be utilized over a range of motion variables and flight conditions. Flight motions could then be predicted by solving the aircraft's equations of motion independently of the flowfield computations.

In the case of motions in the linear aerodynamic regime, the modeling approach has led to the use of the concepts of a linear indicial response and of linear stability derivatives. In a series of papers (see Ref. 7 for a comprehensive review), Tobak and Schiff have shown how these concepts could be extended in a rational way into the nonlinear aerodynamic regime. Their analysis suggests that the nonlinear aerodynamic response to an arbitrary motion of an aircraft can be modeled from knowledge of the aerodynamic responses to a limited number of specified *characteristic* motions. In Ref. 4, Chyu and Schiff employed computational results for two-dimensional unsteady flowfields to validate these

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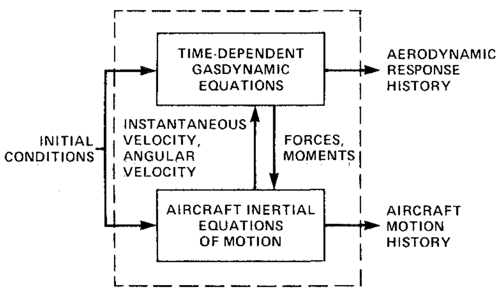
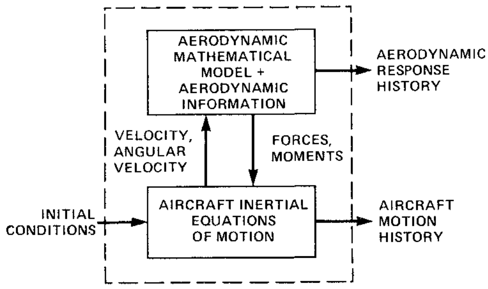


a) Coupled gasdynamic and inertial equations approach.



b) Aerodynamic mathematical modeling approach.

Fig. 1 Approaches to determining the aerodynamic response and aircraft motion histories.

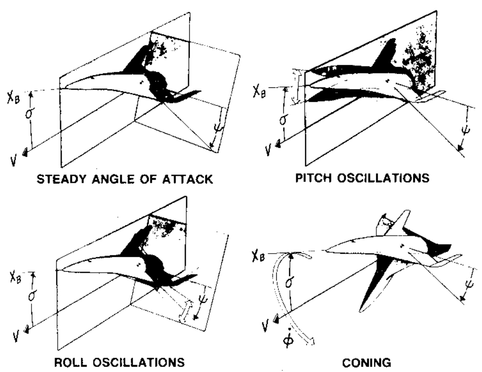


Fig. 2 Aerodynamic axis system and characteristic motions obtained assuming linear variations of the response with the motion rates.

nonlinear modeling concepts for a case involving a single-degree-of-freedom motion, namely, that of a freely deflecting flap attached to a stationary airfoil and immersed in a transonic flow.

In this paper we employ an analogous computational approach to investigate the ability of the aerodynamic modeling technique to describe the aerodynamic contributions in a case involving multiple degrees of freedom, namely, those of a delta wing maneuvering at large angles of attack. Although computation of three-dimensional, unsteady, *viscous* flows by means of the Reynolds-averaged Navier-Stokes equations is not yet feasible, computation of three-dimensional, unsteady, inviscid flows based on the unsteady potential equations is currently possible. In this work we employ a nonlinear vortex-lattice method to evaluate the aerodynamic data in terms of the characteristic motions called for by the aerodynamic model. Time histories of the aerodynamic force and moment responses to a prescribed complex motion are then generated by means of the model and the previously evaluated aerodynamic data. These force and moment histories are compared to histories that are, in principle, exact within the framework of the underlying unsteady potential equations, namely, those obtained by applying the vortex-lattice method directly to the prescribed complex motion. Use of the same numerical technique both to evaluate the aerodynamic coefficients in terms of the characteristic motions and to evaluate the aerodynamic responses to the prescribed complex motion ensures that the unsteady responses are treated consistently in both approaches. Validity of the mathematical modeling approach for the delta wing maneuvering in the high-angle-of-attack regime is demonstrated if the force and moment time histories obtained from the two approaches are in close agreement.

Aerodynamic Mathematical Model

In a series of papers (Refs. 7-14), Tobak and his colleagues have been engaged in developing a mathematical model of the aerodynamic contribution within a framework broad enough to incorporate the variety of aerodynamic phenomena encountered by aircraft maneuvering in the high-angle-of-attack regime. These phenomena include aerodynamic hysteresis, vortex asymmetry and breakdown, large-scale unsteadiness, and dynamic stall (cf. Fig. 4 of Ref. 14). In this study we investigate the applicability of the model, within a more restricted framework, to describe the nonlinear aerodynamics of a maneuvering delta wing. As employed here, the model is subject to the following restrictions: 1) the long-term aerodynamic response to a steady motion is itself steady, 2) the response is a single-valued (although allowably nonlinear) function of the orientation of the body, and 3) the responses are linearly dependent on the motion rates. Restriction 1 rules out the possibility of modeling time-dependent aerodynamic responses, that is, periodic, quasiperiodic, or chaotic responses to a steady motion, such as are seen, for example, in the time-varying flow about a circular cylinder at $Re > 50$. Recent modeling efforts (see Ref. 14), based on the concept of Fréchet differentiability of the aerodynamic response, include treatment of such time-varying responses. Restriction 2 precludes modeling the bivalued aerodynamic responses characterizing static aerodynamic hysteresis, which have been observed in cases of vortex asymmetry on slender bodies of revolution and on slender delta wings. However, hysteresis can also be accommodated within the model (see Ref. 7, and more recently Ref. 14). The third restriction limits the applicability of the model to slowly varying aircraft motions and precludes modeling nonlinear dependence of the aerodynamic responses on the motion rates. Such nonlinear variation with rates has been observed experimentally (as shown, for example, in Ref. 15 for the case of airplane spin motions). Although not included in the model discussed here, nonlinear rate dependence can be easily incorporated in the aerodynamic model.[7,14]

The form that the aerodynamic mathematical model takes is dependent on the particular set of variables used to describe the motion. In this article we employ an aerodynamic axis system, and specify the orientation of the body in terms of pitch and roll coordinates. In the aerodynamic axes, the resultant angle of attack σ is defined as the angle between the body-fixed longitudinal X_B axis and the flight velocity vector (Fig. 2). The plane containing σ is called the resultant-angle-of-attack plane. The roll, or bank, angle ψ is the angle between the normal to the resultant-angle-of-attack plane and a body-fixed axis (normal to the X_B axis), which lies in the plane of the wing. Subject to the above-mentioned restrictions, and under the additional constraint that the vehicle's center of mass follow an essentially rectilinear path (i.e., no lateral plunging), the resulting nonlinear formulation for the pitching-moment coefficient is[7]:

$$
\begin{aligned}
C_m(t) = {} & C_m(\infty;\sigma(t),\psi(t)) + \frac{\dot{\psi} b}{2U} C_{m_{\dot{\psi}}}(\sigma(t),\psi(t)) \\
& + \frac{\dot{\sigma} b}{2U} C_{m_{\dot{\sigma}}}(\sigma(t),\psi(t)) + \frac{\dot{\phi} b}{2U} C_{m_{\dot{\phi}}}(\infty;\sigma(t),\psi(t))
\end{aligned}
\tag{1}
$$

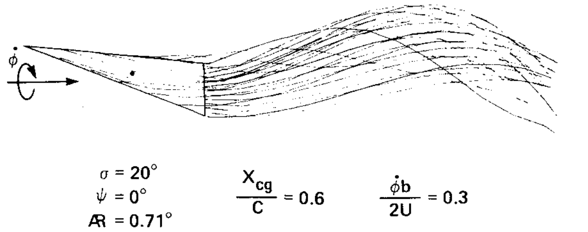


Fig. 3 Computed wake roll-up pattern behind a delta wing undergoing a coning motion.

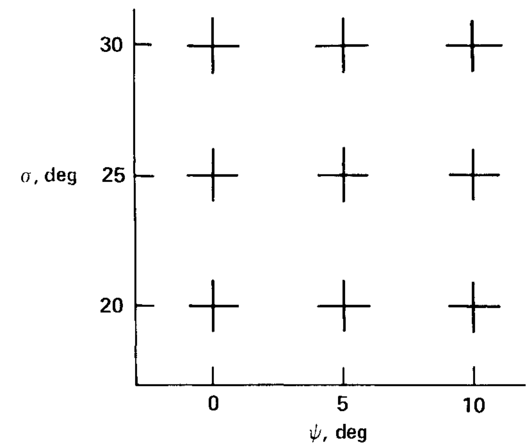


Fig. 4 Range of angle of attack σ and roll angle ψ considered in determining aerodynamic coefficients for the mathematical model, Eq. (1).

Analogous expressions for the yawing- and rolling-moment coefficients C_n and C_l, and for the axial-, side-, and normal-force coefficients C_X, C_Y, and C_N are obtained by substituting these coefficients wherever C_m appears in Eq. (1). Again, the model applies to slowly varying motions of the aircraft although the values of σ and ψ may be large. The mathematical model is seen to contain four terms, and each term can be identified with a specific, characteristic motion from which it may be evaluated. Thus, $C_m(\infty;\sigma(t),\psi(t))$ is the pitching-moment coefficient that would be evaluated from a steady flow with σ and ψ held fixed at $\sigma(t)$, $\psi(t)$. The second term, $C_{m_{\dot\psi}}$, is the contribution to the pitching-moment coefficient due to rolling motion and can be evaluated from small-amplitude oscillations in ψ about $\psi=$const, with σ held fixed and $\dot\phi$ fixed at zero. Similarly, the term $C_{m_{\dot\sigma}}$ is the contribution to the pitching-moment coefficient due to pitching motions and can be evaluated from small-amplitude planar oscillations in σ about $\sigma=$const, with ψ held fixed and $\dot\phi$ fixed at zero. The last term, $C_{m_{\dot\phi}}$, is the rate of change of the pitching-moment coefficient, with coning-rate parameter $\dot\phi b/2U$ evaluated at $\dot\phi=0$, that would be determined from a steady coning motion with σ, ψ, and $\dot\phi=$const. These characteristic motions are illustrated in Fig. 2.

The utility of the aerodynamic modeling approach depends on the ability of the model to encompass the aerodynamic phenomena that occur in flight. In applying a model analogous to Eq. (1), the general aircraft flight motion is decomposed into a sum of characteristic motions. The aerodynamic response to the general motion is modeled as a sum of responses to the characteristic motions. The actual response to the flight motion will differ from the modeled response if aerodynamic phenomena excluded in the development of the model are present. The assumptions made in developing Eq. (1), that the aerodynamic responses are continuous, single-valued functions of the motion variables, restricts the model to cases in which neither hysteresis nor time-dependent aerodynamic bifurcations occur. Within these restrictions, the remaining causes for failure of the model to predict a general response would be either 1) significant nonlinear dependence of the aerodynamic responses on rates of motion within the range of rates actually experienced in flight, or 2) presence of significant interactions between responses to pairs of characteristic motions. Examples of such interactions include those between responses to pitch oscillations and coning motion or between responses to roll oscillations and coning motion, i.e., terms such as $C_{m_{\dot\sigma\dot\phi}}$ or $C_{m_{\dot\psi\dot\phi}}$, which have been excluded in deriving Eq. (1).

Numerical Method

Validation of the mathematical modeling concepts described above requires evaluation of the aerodynamic coefficients in terms of the characteristic motions. The development of a vortex-lattice method[16,17] for unsteady flows has now made feasible the computational evaluation of the aerodynamic coefficients for simple wings maneuvering in the high-angle-of-attack regime. As a test case for the evaluation of the mathematical modeling, three-dimensional maneuvers of a sharp leading-edge delta wing were investigated.

The vortex-lattice method (VLM) is applied to solve the unsteady potential flowfield equations. Solutions describe the evolution of the flowfield around a maneuvering wing initially at rest in the fluid. In the computational procedure, the wing's surface is divided into a number of bound vortex panels. At each time step during the computation, the strengths of the bound panels are determined to enforce the boundary condition that there be no flow through the solid wing. Information describing the wing's maneuver enters the computation through the solid-surface boundary condition. The time evolution of the wake behind the wing is modeled by allowing vortex panels to shed from the trailing edge at each time step. These wake panels have fixed strength and, upon leaving the wing, move with the local fluid velocity.

When the wing is maneuvering at high angles of attack, flow separates near the wing leading edges, and the separated fluid rolls up above and behind the wing to form concentrated vortices. For a sharp-edged wing, this separation line is essentially fixed at the leading edge and does not vary with changes in Reynolds number. Leading-edge separation is modeled in the vortex-lattice method in a manner analogous to that of trailing-edge separation, that is, by allowing vortex panels to shed from specified lines of separation fixed at the leading edge and permitting these panels to move with the local flow velocity. Specification of the strengths of the bound vortices and the positions of the free vortices yields the pressure distribution on the wing and, in turn, the instantaneous airloads. Although the equations solved by means of the numerical method are linear, allowing the vortex panels to roll up in the manner described enables the prediction of nonlinear, unsteady aerodynamic behavior (cf. Refs.16-20). The ability of vortex-lattice methods to enable prediction of the nonlinear aerodynamic forces associated with leading-edge separation at high angles of attack (excluding cases in which vortex breakdown occurs) is also demonstrated in Refs. 19-21. Further, the ability of vortex-lattice methods to enable accurate prediction of unsteady nonlinear airloads is shown in Refs. 22 and 23. Details of the present method are described in Ref. 17. Results of a typical computation, showing the roll-up of the wake of a wing in a coning motion, are shown in Fig. 3.

Method of Model Validation

The procedure for validating the model for the case of the maneuvering delta wing has three major phases:

1) Evaluate the aerodynamic data from vortex-lattice computations in terms of the characteristic motions called for by the model.

2) Generate time histories of the aerodynamic force and moment responses to a prescribed complex motion by means of the aerodynamic mathematical model and the aerodynamic data evaluated in phase 1.

3) Compare the histories obtained in phase 2 with force and moment histories that are, in principle, exact within the framework of the vortex-lattice method of computing the aerodynamic data, namely, those obtained by applying the method directly to the identical complex motion.

Demonstration of the validity of the mathematical modeling approach, as applied to the delta wing maneuvering in the high-angle-of-attack regime, hinges on finding close agreement between the force and moment time histories obtained from the two approaches.

Calculation of Response to Characteristic Motions

The particular wing investigated was a sharp-edged slender delta wing having an aspect ratio of unity (leading-edge sweep angle = 75.96 deg). The center of mass was fixed at the wing half-chord ($X_{cg}/c = 0.50$). The values of the resultant angle of attack and roll angle investigated are shown in Fig. 4. The resultant angle of attack σ ranged from 20 to 30 deg, while the roll angle ψ ranged from 0 (wing level) to 10 deg. The dimensionless motion rates $\dot{\sigma}b/2U$, $\dot{\psi}b/2U$, and $\dot{\phi}b/2U$ ranged up to 0.15.

To obtain the aerodynamic data required by the mathematical model, computations were carried out for the wing in each of the four characteristic motions shown in Fig. 2, at each value of resultant angle of attack and roll angle shown in Fig. 4. The steady-state term $C_k(\infty;\sigma(t),\psi(t))$, where C_k denotes any of the force or moment coefficients, was obtained from a computation in which the resultant angle of attack and roll angle were held fixed, and the flowfield was allowed to evolve until it reached a steady state. In an analogous manner the term $C_{k_{\dot{\phi}}}(\infty;\sigma(t),\psi(t))$ was obtained from a series of computations for steady coning motion in which the resultant angle of attack, roll angle, and coning-rate parameter were fixed and the flowfield was allowed to evolve to a steady state. Note that to an observer fixed in the moving wing, the flowfield due to a steady coning motion is indeed time-invariant. The coefficient was then determined from the observed rate of change of the moment with coning-rate parameter $\partial C_k/\partial(\dot{\phi}b/2U)$ evaluated at $\dot{\phi} = 0$.

The aerodynamic coefficient due to pitch oscillations, $C_{k_{\dot{\sigma}}}(\sigma(t),\psi(t))$, was evaluated from small-amplitude harmonic pitch oscillations about the mean values of resultant angle of attack and roll angle shown in Fig. 4. The wing was specified to move according to

$$\sigma = \sigma_0 + \sigma_1 \sin(\omega_1 t)$$
$$\psi = \psi_0$$
$$\dot{\phi} = 0 \tag{2}$$

The amplitude of the harmonic motion, σ_1, was specified to be less than 2 deg. The aerodynamic damping coefficient was evaluated from the component of the aerodynamic response that was 90 deg out of phase with the wing motion. The rationale is easily seen by substituting the conditions describing the pitch oscillations, Eq. (2), into the aerodynamic model, Eq. (1), to obtain [after a Taylor-series expansion about $\sigma = \sigma_0$ and omission of terms of $O(\sigma_1^2)$]

$$C_k(t) = C_k(\infty;\sigma_0,\psi_0) + \sigma_1 \sin(\omega_1 t)\frac{\partial C_k(\infty;\sigma_0,\psi_0)}{\partial \sigma} + \frac{\omega_1 \sigma_1 b}{2U}\cos(\omega_1 t) C_{k_{\dot{\sigma}}}(\sigma_0,\psi_0) \tag{3}$$

The coefficients in Eq. (3) were obtained from a Fourier integration of the response over one cycle of the motion, as shown for the normal-force coefficient response in Fig. 5. Thus,

$$C_k(\infty;\sigma_0,\psi_0) = \frac{1}{2\pi}\int_0^{2\pi} C_k(t)\,\mathrm{d}(\omega_1 t) \tag{4}$$

$$C_{k_\sigma}(\infty;\sigma_0,\psi_0) = \frac{1}{\pi\sigma_1}\int_0^{2\pi} C_k(t)\sin(\omega_1 t)\,\mathrm{d}(\omega_1 t) \tag{5}$$

$$C_{k_{\dot{\sigma}}}(\sigma_0,\psi_0) = \frac{2U}{\pi b \omega_1 \sigma_1}\int_0^{2\pi} C_k(t)\cos(\omega_1 t)\,\mathrm{d}(\omega_1 t) \tag{6}$$

The steady-state coefficient and its slope can be obtained either from the computations of the oscillatory motion [Eqs. (4) and (5)] or, preferably, from the computations of the steady motion described earlier.

Analogously, the coefficient due to roll oscillations, $C_{k_{\dot{\psi}}}(\sigma(t),\psi(t))$, was evaluated for small-amplitude harmonic roll motions, where

$$\sigma = \sigma_0$$
$$\psi = \psi_0 + \psi_1 \sin(\omega_2 t) \tag{7}$$
$$\dot{\phi} = 0$$

The roll damping coefficient was obtained from

$$C_{k_{\dot{\psi}}}(\sigma_0,\psi_0) = \frac{2U}{\pi b \omega_2 \psi_1}\int_0^{2\pi} C_k(t)\cos(\omega_2 t)\,\mathrm{d}(\omega_2 t) \tag{8}$$

The results of the computations for the characteristic motions are shown in Figs. 6-8. Generation of these diagrams of the aerodynamic coefficients required 36 individual computations, 1 for each of the 4 characteristic motions at the 9 combinations of resultant angle of attack and roll angle shown in Fig. 4. In these computations the wing planform was represented by 5 longitudinal panels, for a total of 30 bound vortex panels on the wing (cf. Refs. 16 and 17). Resolution at this scale required computation times of 60 s/case on a Cray-1

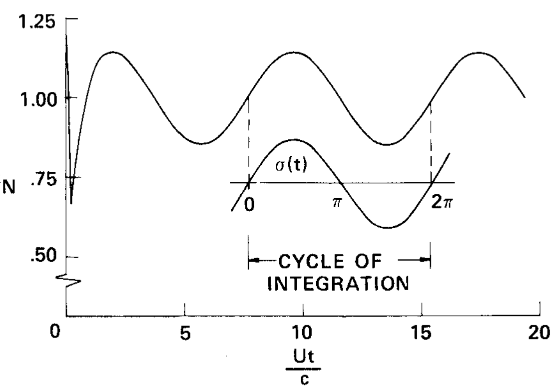


Fig. 5 Fourier integration of normal-force response to evaluate the aerodynamic coefficients from Eqs. (4-6).

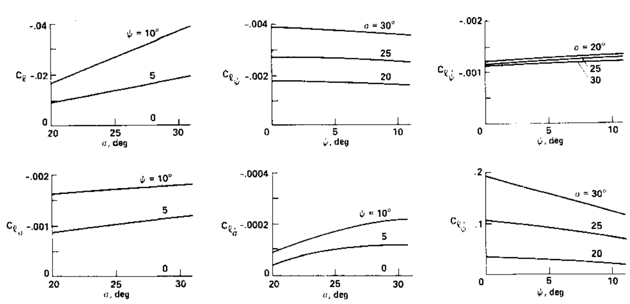


Fig. 6 Rolling-moment coefficients of delta wing evaluated from characteristic motions.

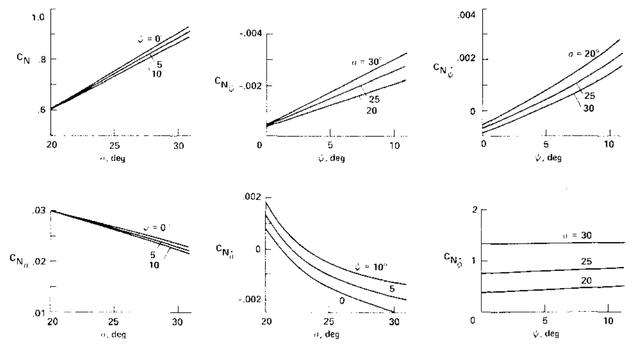


Fig. 7 Normal-force coefficients of delta wing evaluated from characteristic motions.

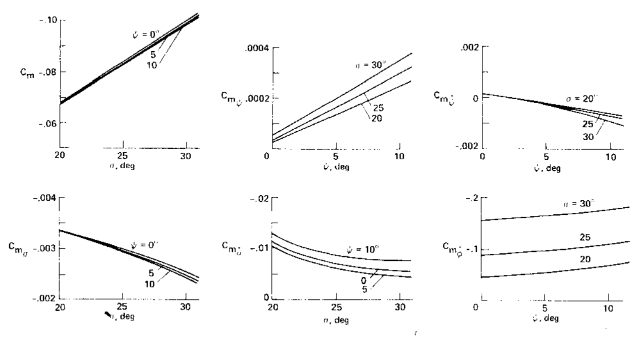


Fig. 8 Pitching-moment coefficients of delta wing evaluated from characteristic motions.

computer. Test computations made with finer grids indicated that the resolution employed was adequate for the purpose of validating the mathematical model.

The computed results are consistent with the assumptions of the mathematical model considered: Over the range of angles of attack, roll angles, and motion rates considered, the aerodynamic responses to the characteristic motions are linearly dependent on the motion rates and are single-valued functions of the angles. Thus, barring the presence of significant nonlinear interactions between the responses to pairs of characteristic motions, the aerodynamic model, Eq. (1), should prove to be valid. In general, the results for the computed aerodynamic terms are in good agreement with experimentally measured results for cases in which no vortex breakdown is present. However, from the standpoint of accuracy, the roll-damping coefficient $C_{l\dot{\psi}}$ should probably have a positive value at $\psi = 0$, as discussed in Ref. 24. Also, the static normal-force coefficient shown in Fig. 7 should have been slightly larger to agree with experimentally measured results for sharp-edged delta wings. Closer agreement between the computed and experimental results could be obtained by using a finer computational resolution. Since the purpose of this study was to investigate the validity of the mathematical model, not the accuracy of the vortex-lattice method, the additional computational effort was not deemed warranted. To investigate the validity of the modeling method, it was sufficient that the identical numerical method and panel resolution be employed to compute both the aerodynamic responses to the prescribed motions and to the characteristic motions.

Calculation of Response to Prescribed Complex Motions

The prescribed complex motions combined pitch oscillations, roll oscillations, and coning motion. The combined motions all had the basic form

$$\sigma = \sigma_0 + \sigma_1 \sin(\omega_1 t), \qquad \dot{\sigma} = \omega_1 \sigma_1 \cos(\omega_1 t)$$

$$\psi = \psi_0 + \psi_1 \sin(\omega_2 t), \qquad \dot{\psi} = \omega_2 \psi_1 \cos(\omega_2 t)$$

$$\phi = \omega_3 t, \qquad \dot{\phi} = \omega_3 \tag{9}$$

Aerodynamic response histories of the pitching-moment, rolling-moment, and normal-force coefficients were computed from Eq. (1), with the aerodynamic coefficients obtained from tables of the data shown in Figs. 6-8, and values of $\dot{\sigma}$, $\dot{\psi}$, and $\dot{\phi}$ obtained from Eq. (9). The aerodynamic responses to the prescribed motions were also obtained from direct VLM computations. As was mentioned earlier, the use of the identical vortex-lattice method to evaluate both the nonlinear responses to the characteristic motions and the responses to the prescribed complex motions ensures a consistent treatment of the time-history effects. Thus discrepancies, if present, between results obtained using the modeling approach and those obtained from the direct VLM computations must be attributed to the inadequacy of the aerodynamic model.

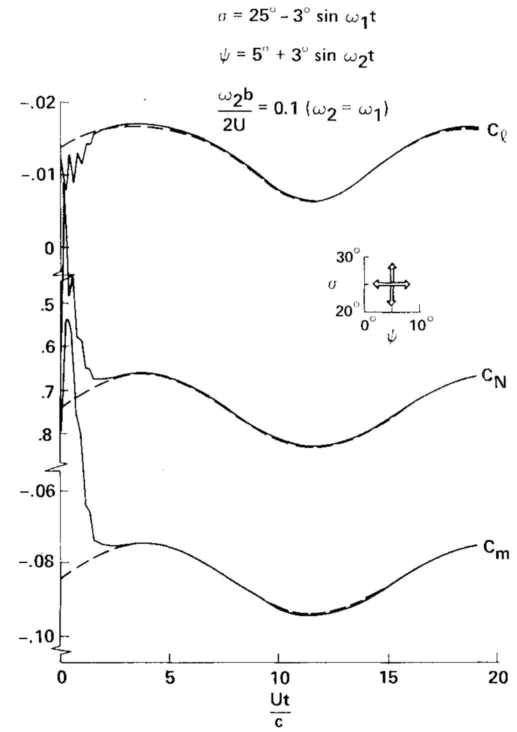


Fig. 9 Aerodynamic response of delta wing to combined pitch and roll oscillations.

Validation of the Mathematical Model

The computed responses to the prescribed motions are illustrated in Figs. 9-14. Responses of the pitching-moment, rolling-moment, and normal-force coefficients to the prescribed motions are shown in Figs. 9-14 as functions of wing-chord lengths of travel Ut/c. In each figure the dotted lines show the histories obtained from the mathematical model, Eq. (1), while the solid lines indicate the results obtained from direct VLM computations. For the direct computations, the overshoot indicated at the beginning of each time history occurs because the motion is started impulsively from rest. It will be recalled that the aerodynamic model considered in this work is obtained under the assumption of slowly varying motions and is not expected to model the impulsive start. If the short initial transient period is excluded, in all cases considered, the aerodynamic responses obtained from the model show reasonable agreement with those obtained from the direct computation.

The motions considered increase progressively in complexity from Figs. 9 to 14 and demonstrate various capabilities of the mathematical modeling concept. Combined pitch and roll oscillations about a (nonconing) steady motion, where $\sigma_0 = 25$ deg and $\psi_0 = 5$ deg, are considered in Figs. 9 and 10. The mo-

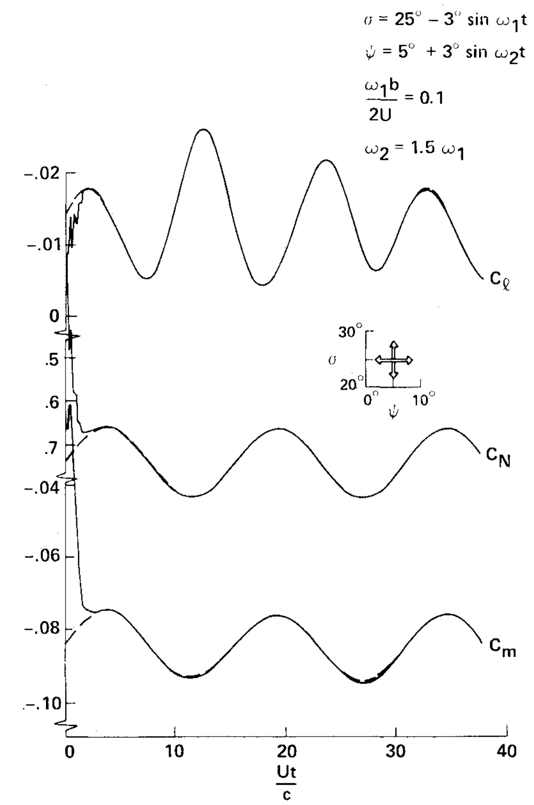


Fig. 10 Aerodynamic response of delta wing to combined pitch and roll oscillations.

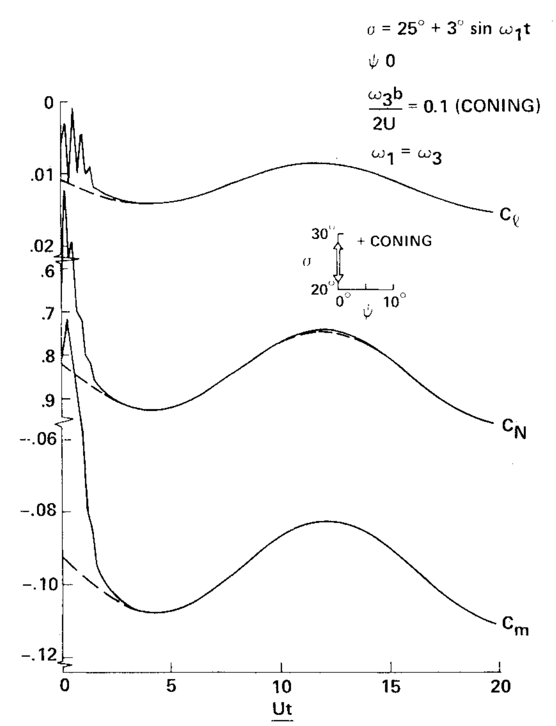


Fig. 11 Aerodynamic response of delta wing to combined pitch oscillations and coning motion.

tions are similar in amplitude, differing only in the frequencies of the various oscillations. The agreement obtained between the results of both sets of computations in these cases demonstrates that the unsteady aerodynamic responses vary linearly with the motion rates, over the range of rates considered, in accord with the assumptions underlying Eq. (1). The motions considered in Figs. 11-14 consist of pitch and roll oscillations superimposed on steady coning motions. Planar

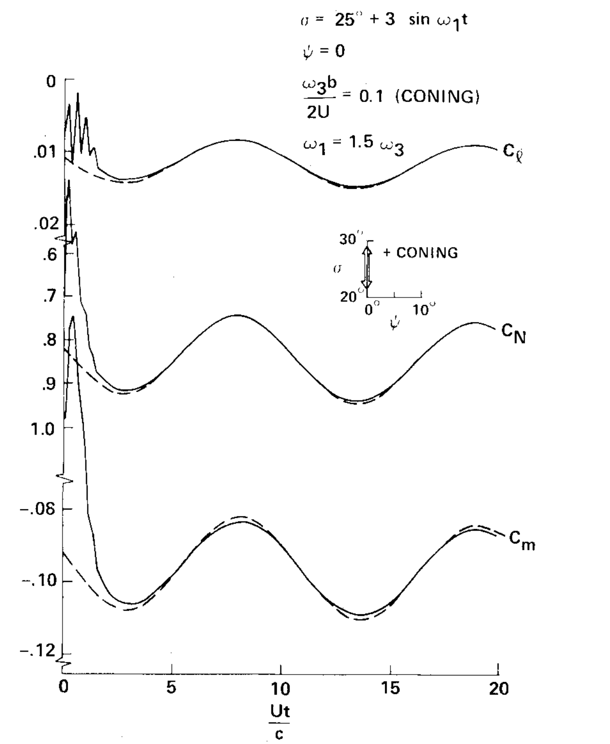


Fig. 12 Aerodynamic response of delta wing to combined pitch oscillations and coning motion.

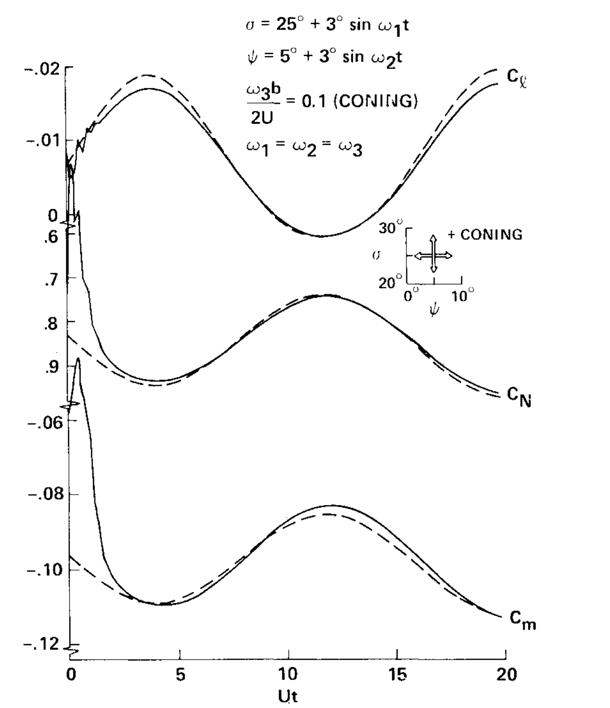


Fig. 13 Aerodynamic response of delta wing to combined pitch oscillations, roll oscillations, and coning motion.

pitch oscillations, having an amplitude of 3 deg and different frequencies, combined with a coning motion, where $\sigma_0 = 25$ deg and $\psi_0 = 0$, are considered in Figs. 11 and 12. The motions considered in Figs. 13 and 14 include both pitch oscillations and roll oscillations superimposed on a steady coning motion, where $\sigma_0 = 25$ deg and $\psi_0 = 5$ deg. Here, the agreement obtained between the results of both sets of computations indicate that no significant interactions existed between responses to pairs of characteristic motions. Further, the agreement provides the first direct confirmation of the modeling concepts for multi-degree-of-freedom motions.

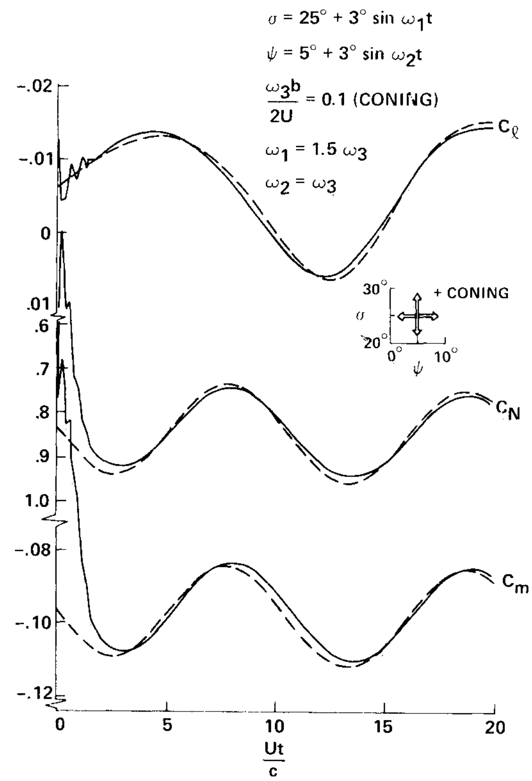


Fig. 14 Aerodynamic response of delta wing to combined pitch oscillations, roll oscillations, and coning motion.

Discussion

As mentioned earlier, the main causes for failure of the aerodynamic model considered in this study would be either 1) significant nonlinear dependence of the aerodynamic responses on the motion rates considered, or 2) presence of significant interactions between responses to pairs of characteristic motions. Thus, the agreement shown in Figs. 9-14 would tend to indicate that, over the range of angles and motion rates considered, the mathematical model presented in Eq. (1) is adequate to describe the aerodynamic response to complex motions of the delta wing. In actuality, for the range of pitch and roll rates considered, the contributions of the pitch-damping and roll-damping terms in Eq. (1) were almost negligible. As a result, the aerodynamic interactions between pairs of the characteristic motions, which are of higher order than the individual damping terms, must be negligible. In these circumstances, it is not surprising that the mathematical model would appear to be validated. Although the VLM computations confirm the validity of the model for the cases considered, the cases themselves do not conclusively demonstrate the limits of the range of motions and rates for which the model is valid. Nevertheless, the procedure discussed supports the validity of the modeling concept and, further, indicates the way in which computational fluid dynamic methods can be used to validate a candidate aerodynamic model.

The demonstrated validity of the aerodynamic mathematical modeling approach in the case of the (limited) delta-wing maneuvers supports our belief that an analogous mathematical model will be applicable to describe the nonlinear response of aircraft over a range of high-angle-of-attack flight maneuvers. This is fortunate, since the modeling approach permits much more economical evaluation of large numbers of flight trajectories in comparison with the coupled-equations approach. Flowfield data, once evaluated in terms of the characteristic motions, can be summarized in tables similar to those constructed from Figs. 6-8 and reutilized for all computations of flight motions. Also, the modeling concept is compatible with established analytical methods for evaluating the stability of motions about their equilibrium conditions. Utilizing analytic methods will reduce the number of trajectories to be computed and thus further reduce the required computational effort. As a result, the nonlinear modeling approach would appear to be the method of choice in the design of flight control systems and in flight simulations.

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